KATHOLIEKE UNIVERSITEIT LEUVEN

Statistics and Econometrics Seminar

Joint organization by
ORSTAT, Faculty of Business and Economics and the Statistics Research Group,
Faculty of Science
Leuven Statistics Research Center

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"Asymptotic normality of Support Vector Machines"

Thursday, March 24, 2011
12.00–13.00h

Location: Room HOG 03.101, Naamsestraat 69, Leuven.
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Abstract. In nonparametric classification and regression problems, support vector machines (SVMs) recently attract much attention in theoretical and in applied statistics. In an abstract sense, SVMs can be seen as M-estimators for a parameter in a (typically infinite dimensional) reproducing kernel Hilbert space. Estimating a (completely unknown) regression function is burdened with the problem of ill-posedness: arbitrarily small changes can have serious effects on estimating the regression function. As a consequence, there is no uniform rate of convergence (learning rate), and no estimator can be simultaneously universally consistent and qualitatively robust. For a fixed (arbitrarily small) regularization parameter λ ¿ 0, the "theoretical SVM" $f_{L,P,\lambda}$ is a smoother approximation of the unknown regression function. Estimating this approximation instead of the regression function is well-posed so that the just mentioned problems do not appear for this modified estimation problem. After a short introduction into the theory and recent results on SVMs, it is shown that estimating $f_{L,P,\lambda}$ by the "empirical SVM" $f_{L,\mathbf{D_n},\lambda_{\mathbf{D_n}}}$ is asymptotically normal with rate $\sqrt{n}$. That is, the standardized difference $\sqrt{n}(f_{L,\mathbf{D_n},\lambda_{\mathbf{D_n}}} - f_{L,P,\lambda})$ converges weakly to a Gaussian process in the reproducing kernel Hilbert space. This is done by an application of the functional delta-method and by showing that the SVM-functional $P \mapsto f_{L,P,\lambda}$ is suitably Hadamard-differentiable.